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Optimal Placement of PMUs for Power System Observability Using Topology Based Formulated Algorithms

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Abstract: This study presents optimal placement of Phasor Measurement Units (PMUs) for the purpose of power system observability using topology based formulated algorithms. The optimal PMU placement problem is formulated to minimize the number of PMUs installation subject to full network observability. The Branch and Bound (B and B) and genetic algorithm optimization methods are selected to solve the problem, which are suitable for problems with integer and Boolean variables. Topology-based algorithm used for observability analysis and a hybrid method of topology transformation and nonlinear constraint is used to form constraints. The IEEE 14, 30, 57 and 118-bus and the New England 39-bus test systems are used for simulation purposes. The results are compared with those of previously implemented. Results show that with installing PMUs on less than 25% of system buses all of the system can be made observable.

Key words: Power system observability, phasor measurement unit, optimal PMU placement, branch and bound algorithm, genetic algorithm

INTRODUCTION

Observability of power system is a necessary condition for state estimation. State estimation provides estimation for all metered and unmetered electrical quantities of the power system. Its output is used for online operation and management of power system. Power system observability analysis reveals if the present set of measurements and their distribution are sufficient to solve the current state of the power system. Traditional state estimators use Weighted Least Squares (WLS) method to solve system state estimation problem with conventional measurements such as voltage magnitudes, bus real and reactive power injections and line real and reactive power flows. This is a nonlinear-iterative method and its solution time is considerably more than linear methods. With development of digital signal processing and global positioning systems, Phasor measurement system was introduced in last 1980s. Because of the ability of PMUs to gather synchronized phasors of voltages and currents from all over the system, they have been used in various fields of modern power systems studies such as state estimation, transient stability analysis, fault location studies, wide area protection and small signal stability (Baldwin *et al*., 1993; Phadke and Thorp, 2006; Kai *et al*., 2006; Zhao *et al*., 2005).

Since, PMU measures the phase and amplitude of installed bus voltages and all connected branches currents, there is no need to install PMU in all buses, thus a suitable approach is needed to determine the optimum set and locations of PMUs in the electric network.

Lots of interests have been shown toward optimal PMU placement in recent years. For first time optimal PMU placement problem is formulated using graph theoretical observability analysis (Baldwin *et al*., 1993). Zhao *et al*. (2005) dealt with the placement of a minimal set of PMUs to make system completely observable and collected more valuable dynamic data of power system at the same time using simulated annealing optimization method. PMU placement considering complete and incomplete observability is done using tree search method (Nuqui and Phadke, 2005).

Gou (2008a, b) had presented a generalized integer linear programming formulation and solution approach for PMU placement. (Chakrabarti and Kyriakides, 2008) used a exhaustive binary search method to find the minimum number of PMUs for power system complete observability.

Phasor Measurement Unit (PMU) placement for some other purposes such as voltage stability and transient stability studies were also carried out by Mili *et al*. (1990). The minimal PMU placement was carried out to make observable the transmission network fault location (Kai *et al*., 2006).

The PMU placement problem is formulated and solved using several optimization methods and algorithms such as graph theoretic procedure and Dual Search (Baldwin *et al*., 1993) Tabu search (Jiangnan *et al*., 2006), integer programming (Bei and Abur, 2004), Nondominated sorting genetic algorithm (Milosevic and Begovic, 2003), integer linear programming (Gou, 2008a, b; Dua *et al*., 2008). The main drawback of integer programming is that it may be trapped in local minima as mentioned by Chakrabarti and Kyriakides (2008).

MATERIALS AND METHODS

Observability analysis is a fundamental component of real-time state estimation. There are two major algorithms for power network observability analysis: topology based algorithms and numerical methods. Topology methods use the decoupled measurement model and graph theory. In these methods decision is based on logical operations. Thus, they require only information about network connectivity, measurement types and their locations. If a full rank spanning tree can be constructed with current measurement set, the system will be observable.

Numerical methods, on the other hand, use either fully coupled or decoupled measurement models. These methods are based on numerical factorization of the measurement Jacobian or measurement information gain matrix. If any of these matrices is full rank, the system will be observable and there will be a unique solution for state estimation problem. Numerical methods are not suitable for large systems because they are involved with huge matrix manipulation and have their own computational complexity. Therefore, topology based method is used in this paper. It should be mentioned that OPP problem is formulated with numerical method based genetic algorithm, hitherto.

TOPOLOGY BASED FORMULATION

In this study, the optimal PMU placement problem is described as finding a scheme with minimal PMUs and their installation locations such that the entire system becomes observable. The used observability rules are as follows:

- For PMU installed buses, voltage phasor and current phasor of all its incident branches are known. These are called as direct measurements
- If voltage and current phasors at one end of a branch are known then voltage phasor at the other end of the branch can be obtained. These are called pseudo measurements
- If voltage phasors of both ends of a branch are known then the current phasor of this branch can be obtained directly. These measurements are also called pseudo measurements
- For a zero-injection bus i in a N-bus system we have:

$$\sum_{j=1}^{N} Y_{ij} V_j = 0 \tag{1}$$

where, Y_{ij} is the ij-th element of admittance matrix of the system and V_j is the voltage phasor of j-th buse.

Therefore, if there is a zero-injection bus without PMU whose incident branches current phasors are all known but one, then the current phasor of the unknown one could be obtainable using KCL equations.

- If there is a zero-injection bus with unknown voltage phasor and voltage phasors of its adjacent buses are all known, then the voltage phasor of the zero-injection node can be found by node equations
- If there exists a group of adjacent zero-injection buses whose voltage phasors are unknown but the voltage phasors of all adjacent buses to the group are known, then the voltage phasors of zero-injection buses can be obtained through node equations

The measurements obtained from rules 4-6 are called extended measurements.

BRANCH AND BOUND (B AND B) APPROACH

The OPP formulation based on topological observability method that used in B and B approach is as follows:

$$\begin{aligned} &\mathrm{Min} \sum_{i=1}^{N} w_i x_i \\ &\text{Subject to } \; F(X) \geq 1 \quad (f_i(X) \geq 1 \quad i = 1, 2, \ldots, N) \end{aligned} \tag{2}$$

where, N is total No. of system buses, w_i is weight factor accounting to the cost of installed PMU at bus i, X is a binary variable vector whose entries are defined as Eq. 3 and F(X) is a vector function that its entries are non-zero if the corresponding bus voltage is observable using the given measurement set and according to observability rules mentioned in earlier; otherwise its entries are zero.

$$x_i = \begin{cases} 1 & \text{if a PMU is needed at bus i} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Two different methods have been proposed to take into account the zero-injection buses. These are topology transformation and nonlinear constraint functions methods. In this study, constraint functions considering adjacent zero-injection buses are constructed using a novel hybrid topology transformation-nonlinear constraint method. This method is illustrated below using an example.

In the New England 39-bus test system (Fig. 1), buses 5 and 6 are zero-injection buses. First, these two buses are combined together to make the new bus 5' with the new constraint function (topology transformation):

$$f_{5'} = x_{5'} + x_4 + x_7 + x_8 + x_{31} + x_{11} \tag{4}$$

Next, the new bus constraint function is eliminated from the formulation using nonlinear constraint method. Thus, the neighboring buses constraint functions are corrected as follows:

$$\begin{aligned}
f_4 &= x_4 + x_{5'} + x_3 + x_{14} + f_{5'} \cdot f_7 \cdot f_8 \cdot f_{31} \cdot f_{11} \\
f_7 &= x_7 + x_{5'} + x_8 + x_4 \cdot f_{5'} \cdot f_8 \cdot f_{11} \cdot f_{31} \\
f_8 &= x_8 + x_{5'} + x_7 + x_9 + f_4 \cdot f_{5'} \cdot f_7 \cdot f_{11} \cdot f_{31} \\
f_{11} &= x_{11} + x_{5'} + x_{12} + x_{10} + f_4 \cdot f_{5'} \cdot f_7 \cdot f_8 \cdot f_{31} \\
f_{31} &= x_{31} + x_{5'} + f_4 \cdot f_{5'} \cdot f_7 \cdot f_8 \cdot f_{11}
\end{aligned} \tag{5}$$

Branch and bound is an optimization method suitable for non-convex problems and works very well for solving OPP. In this study, the problem is spitted into sub-problems (equal to the number of nodes) and for each node the upper and lower bounds are calculated using relaxation methods. Whenever the lower bound of a node becomes larger than its upper bound, that node is removed from the tree. In this case, in each step, each sub-problem is divided into two sub-trees according to fixing x_i to either zero or one and a sub-tree which has the lowest lower bound is selected to continue optimization. In the next step, a variable having the value which is closest to either 0 or 1 will be selected to be fixed.

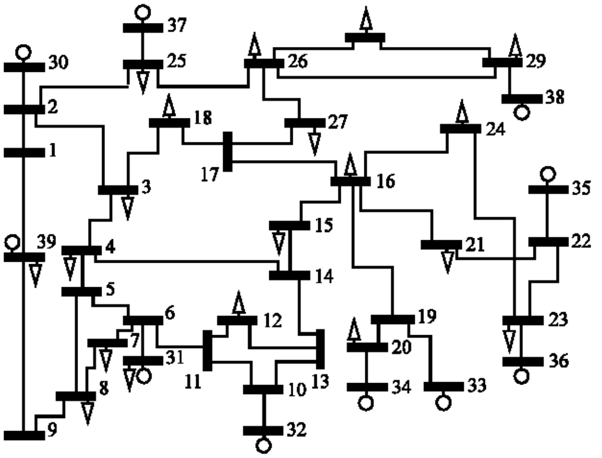

Fig. 1: New England 39-bus test system

GENETIC ALGORITHM APPROACH

Genetic algorithm is one of the effective meta-heuristic methods developed in order to solve nonlinear and non-convex optimization problems. This algorithm is not dependent to objective function gradient. Besides, it has the capability of globally convergence. In order to formulate the OPP in this approach, topology of the system is shown by bus connection matrix (A). A is a N_bus×N_bus symmetric binary matrix that it's arrays defined as Eq. 6.

$$A_{ij} = \begin{cases} 1 & \text{if } i = j \text{ or } i \text{ connected to } j \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

In this method, the OPP problem is formulated as follows:

$$\begin{aligned}
&\text{Min} \sum_{i=1}^{N} w_i x_i \\
&\text{Subject to } \sum_{i=1}^{N\_bus} y_{ij} \geq 1 \quad \text{for } j = 1, 2, \ldots, N\_bus
\end{aligned} \tag{7}$$

where, y_{ij} is observability index and defined as:

$$y_{ij} = \begin{cases} x_i & \text{if } A_{ij} = 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for } j = 1, 2, \ldots, N_{\_bus} \tag{8}$$

From observability rules mentioned earlier, it can be conducted that a zero injection bus is observable if all its adjacent buses be observable. Thus, to consider zero injection buses effect, a modified A matrix is developed by merging zero injection buses with one of the adjacent buses. In Fig. 2, bus-4 is zero injection bus and it merged with third bus and made bus numbered with 3'.

Figure 3 shows the flowchart of OPP problem solution using genetic algorithm.

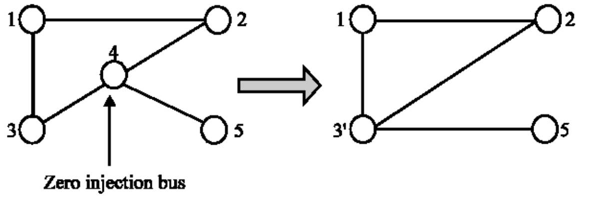

Fig. 2: Zero injection bus merging method

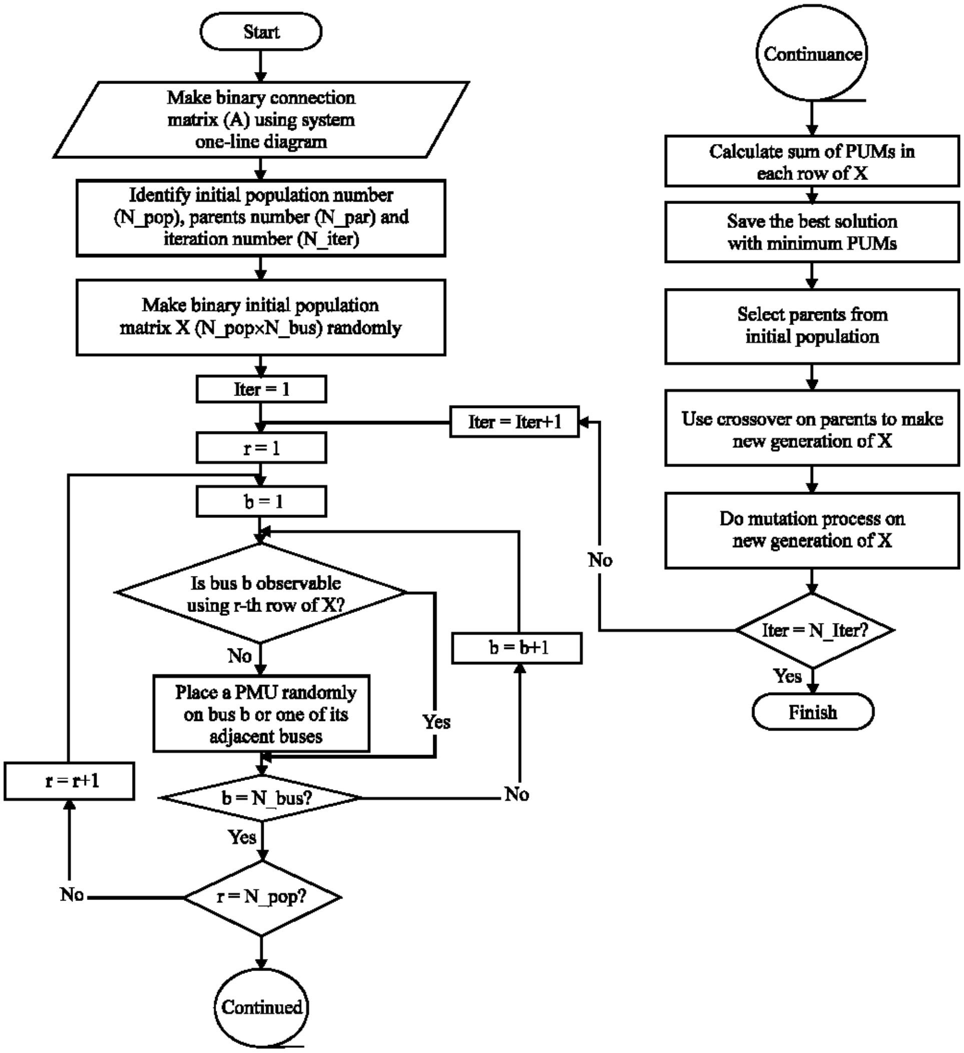

Fig. 3: Flowchart of OPP solution using GA

CASE STUDY RESULTS

The IEEE 14-bus, 30 -bus, 57-bus and 118-bus and the New England 39-bus test systems are used for observability analysis studies using Branch and Bound (B and B) and Genetic Algorithm (GA) approaches. Table 1 shows the data for these systems.

Optimum number of PMUs and their location using B and B approach and without considering zero injection buses information are shown in Table 2.

In Table 3, optimum PMU placement is done using GA approach and without considering zero injection buses information.

Optimum number of PMUs and their location using B and B approach and with considering zero injection buses information are shown in Table 4.

Table 1: Test systems data

Test system	No. of branches	No. of ZI buses	Location of ZI buses
14-bus	20	1	7
30-bus	41	5	6-9-11-25-28
39-bus	46	12	1-2-5-6-9-10-11-13-14-17-19-22
57-bus	78	15	4-7-11-21-22-24-26-34-36-37-39-40-45-46-48
118-bus	179	10	5-9-30-37-38-63-64-68-71-81

Table 2: Optimum number and location of PMUs using B and B approach without considering ZI buses information

Test system	No. of PMUs	Location of PMUs
14-bus	4	2-6-7-9
30-bus	10	1-5-6-9-10-12-15-19-25-29
39-bus	13	2-6-9-10-13-14-17-19-22-23-25-29-34
57-bus	16	1-6-9-15-19-22-25-28-32-36-38-41-47-51-53-57
118-bus	32	3-5-9-12-15-17-21-23-28-30-36-40-44-46-51-54-57-62-64-68-71-75-80-85-86-91-94-101-105-110-114

Table 3: Optimum number and location of PMUs using GA approach without considering ZI buses information

Test system	No. of PMUs	Location of PMUs
14-bus	4	2-6-7-9
30-bus	10	1-2-6-9-10-12-15-19-25-27
39-bus	13	2-6-9-10-11-14-17-19-20-22-23-25-29-
57-bus	18	1-6-10-15-19-22-25-27-32-35-37-38-43-46-49-52-55-56
118-bus	33	4-12-17-20-24-27-31-33-36-39-41-46-51-53-62-65-67-73-75-77- 80-85-86-91-92-94-98-100-105-106-110-112-115

Table 4: Optimum number and location of PMUs using B and B approach with considering ZI buses information

Test system	No. of PMUs	Location of PMUs
14-bus	3	2-6-9
30-bus	7	3-5-10-12-19-24-27
39-bus	9	3-9-12-16-23-29-31-34-37
57-bus	12	1-5-9-14-15-20-25-28-32-50-53-56
118-bus	29	3-5-9-12-15-17-20-23-29-34-40-45-49-52-56-62-65-71-75-77-80-85-86-91-94-101-105-110-115

Table 5: Optimum number and location of PMUs using GA approach with considering ZI buses information

Test system	No. of PMUs	Location of PMUs
14-bus	3	2-6-9
30-bus	7	1-2-10-12-15-20-27
39-bus	8	3-8-13-16-20-23-25-29
57-bus	11	1-5-13-19-25-29-32-38-41-51-54
118-bus	29	2-8-11-12-15-19-21-27-31-32-34-40-45-49-52-56-62-65-72-75-77-80-85-86-90-94-101-105-110

Table 6: Comparison of results of different algorithms

Test system	14-bus	30-bus	39-bus	57-bus	118-bus
Branch and bound (proposed)	3	7	9	12	29
Topology based genetic algorithm (proposed)	3	7	8	11	29
Tabu search	3	N/A	10	13	N/A
Integer linear programming	3	N/A	N/A	14	29
Dual search	3	N/A	N/A	N/A	29
Graph theoretic procedure	5	11	N/A	19	38
Nondominated Sorting Genetic Algorithm (NSGA)	3	7	N/A	12	29
Integer programming	3	7	N/A	12	29

N/A: Not available

In Table 5, optimum PMU placement is done using GA approach and with considering zero injection buses information.

The results obtained using proposed approaches are compared with some other algorithms such as Tabu search (Jiangnan *et al*., 2006), Integer linear programming (Gou, 2008), nondominated sorting genetic algorithm (Milosevic and Begovic, 2003), Graph theoretic procedure and Dual Search (Baldwin *et al*., 1993). Table 6 shows the comparison of results. In Table 6, N/A shows that the result was not available for that case.

CONCLUSION

In this study, the problem of optimum PMU placement for power system observability was investigated. The OPP problem was formulated using topology based algorithms and solved using branch and bound algorithm and genetic algorithm, proposed algorithms applied to some standard test systems.

Simulation results on the IEEE 14, 30, 57 and 118-bus test systems and the New England 39-bus test system indicate that the proposed placement methods satisfactorily provides observable system measurements with minimum number of PMUs.

The results show good improvement in decreasing the number of installed PMUs comparing with earlier applied methods.

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