

## Physical and Numerical Modeling of the Nappe Flow in the Stepped Spillways

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**Abstract:** In the present study based upon the dimensional similitude, two models have been developed for calculation of energy dissipation which is correlated with some dimensionless parameters. To find the unknown coefficients of these equations, several physical models have been established in the hydraulic laboratory. Several small and large stepped spillways (with average slope of 15 and 25°) have been installed in the laboratory flume constructed in the hydraulic laboratory, Shahid Chamran University, Iran. Several hydraulic parameters have been measured which are used for optimization of noted unknown coefficients. The results of the developed models have been compared with the observed data and the previous studies results in the literature. The comparative study shows a very good agreement exists between the results of this study with those from observation and reported in the literature. Besides to experimental study, ANSYS has been employed to simulate the experimental data and their related results. This study shows that ANSYS is able to predict results almost the same as experimental findings with less than 6% error.

**Key words:** Stepped, spillways, physical, modeling, ANSYS

### INTRODUCTION

A longitudinal profile of a fall has shown in the Fig. 1. In this figure  $y_c$  is the critical depth,  $h$  is the height of fall,  $y_1$  and  $y_2$  are the conjugate depths,  $L_p$  is the length between fall and the incident point of the jet in the stilling basin,  $Y_p$  is the depth of stilling basin,  $H_t$  is the total energy of the flow upstream of the fall and  $H_1$  and  $H_2$  are the flow energy before and after the hydraulic jump, respectively. The flow, after falling, enters into the pool downstream of the fall. Due to mixing with the water in the pool, a part of total energy  $H_t$  is dissipated. Then, a supercritical flow is established and due to the downstream sub critical flow, the hydraulic jump is formed and as a result, a part of the flow energy is dissipated. A stepped spillway includes several falls (vertical drops) which are connected to each other on the steep slope.

Two forms of the flow including; Skimming and Nappe flows are established over stepped spillways. In the Nappe flow; the flow hits the horizontal part of each step of the spillway consequently from upstream to the downstream. Due to the slope of the drop, the complete hydraulic jump may form or may not. The Nappe flow can be developed in the low flow discharge and large depth of the step. The energy dissipation in the steps is due to the kind of hydraulic jump and mixing of air with the flow water (Chamani and Rajaratnam, 1994). In the Skimming flow, steps act as a large roughness against the flow over

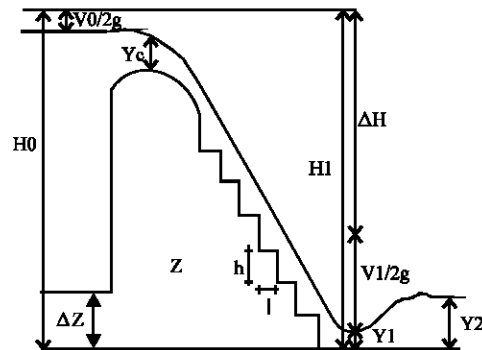


Fig. 1: Longitudinal profile of a stepped spillway

steps. Ohtsu *et al.* (2000) have described more about the flow characteristics in the skimming flow over stepped spillways. In the Skimming flow over steps a pseudo-bottom which connects the end of steps to each other is established. Researchers believed that the secondary flow under this pseudo-bottom is the cause of the most of the energy dissipation in this flow regime over the stepped spillway (Chanson, 1994a, c). The amount of the dissipated energy due to the hydraulic jump is not reported in the literature (Chamani and Rajaratnam, 1994; Chanson, 1994a). This research intends to develop a model for estimation of the energy dissipation in the stepped spillways. The energy dissipation due to hydraulic jump in the low longitudinal slope stepped

spillways is more important than the steep spillways. Due to this reason, several tests over seven stepped spillways with low slopes including 25° (5, 10 and 15 steps) and 15° (5, 10, 15 and 30 steps) have been undertaken. The results of this study have been compared with the other researchers' findings. For instance, Rand (1955) has undertaken several experiments over vertical drops and reported several equations for computation of the flow characteristics. Chanson (1994a) has presented the following equation for the energy dissipation in the vertical drops.

$$\frac{\Delta H}{H_t} = \frac{H_t - H_l}{H_t} = 1 - \frac{y_1}{y_c} - \frac{1}{2} \frac{y_1}{y_c} \left( \frac{y_1}{y_c} \right) \quad (1)$$

In Eq. 1  $H_{Dam}$  = the total head of the weir which is equal to  $N$  multiplied by  $h$  (the height of each step). Equation 1 can be rewritten as function of  $(y_c/h)$  as follows:

$$\frac{\Delta H}{H_t} = 1 - \frac{0.54 \left( \frac{y_c}{h} \right)^{0.275} + 1.715 \left( \frac{y_c}{h} \right)^{-0.55}}{\frac{3}{2} + \frac{H_{Dam}}{y_c}} \quad (2)$$

Equation 2 can be applied to calculate the dissipated energy in the Nappe flow (Chanson, 1994a). Chanson has compared Eq. 2 with findings of Stephenson (1979). It has been shown that there is good agreement between results of Eq. 2 and the experimental results. The relative head loss equation has been expressed as follows (Chamani and Rajaratnam, 1994):

$$\frac{\Delta H}{H_t} = 1 - \frac{\left\{ (1-\alpha)^N \left[ 1 + 1.5 \left( \frac{y_c}{h} \right) \right] + \sum_{i=1}^{N-1} (1-\alpha)^i \right\}}{N + 1.5 \frac{y_c}{h}} \quad (3)$$

Chamani and Rajaratnam (1994), using experimental data of Horner for  $h/l = 0.421$  with 8, 10, 20 and 30 steps;  $h/l = 0.526$  with 10 and 30 steps;  $h/l = 0.736$  with 10 and 30 steps and  $h/l = 0.842$  with 10 and 30 steps have calculated  $\alpha$ . They have shown that  $\alpha$  decrease with increasing  $y_c/l$ . Chanson (1994b) has discussed that for calculating the relative energy loss, Eq. 2 is more reliable than Eq. 3. It does not need the energy loss to be calculated in each step. Several researchers including Ellis (1989), Peyras *et al.* (1991) and Chamani and Rajaratnam (1994) believe that the energy loss in the Nappe flow is more than the Skimming flow. Chanson (1994b, c) believes

that in the long stepped spillways with uniform flow, the energy loss in the Skimming flow is more than the Nappe flow. Chanson and Gonzalez (2004) have reviewed recent advances in stepped spillway design. There is a lack of mathematical modelling of the stepped spillways in the prediction of the stepped spillway head loss in the literature. In this research an attempt has been paid to simulation of the stepped spillway hydraulics using ANSYS 10.0 (2005).

## MATERIALS AND METHODS

This research program has been undertaken in the hydraulic laboratory of Shahid Chamran University, Iran. Laboratory flume was made of glass with 10 mm thickness and its properties are as follows: length = 10 m; width = 25 cm; height = 48 cm. Seven models of stepped spillways with 25° slope (5, 10, 15 steps) and 15° slope (5, 10, 15, 30 steps) were built of galvanized sheet. They were installed 4 m downstream of the upstream wall of the laboratory flume, individually. The water flow was supplied into the flume by a tank with a control valve. The height of the tank is 4.5 m. To control the hydraulic jump, a slide gate was used in the end of the laboratory flume. The location of the hydraulic jump was fixed at 0.5 m downstream of the toe of the stepped spillway. The depth of flow before and after the hydraulic jump was measured with a point gauge (0.1 mm accuracy) in the center of the flume. The discharge of the flow was measured with 53° triangle weir. A large amount of air was entered in the flow before hydraulic jump. Due to two phase flow, there was fluctuation in the water surface and the measurement of the water surface level was involved with 3 mm error. The water surface level immediately after hydraulic jump was measured somewhere downstream which air bubbles were observed.

Head loss due to hydraulic jump over each step is summation of two parts; the head loss between upstream of fall and the section before the hydraulic jump. It is due to impact of the flow jet to the water of pool. Another part of head loss is due to the hydraulic jump which may be a complete hydraulic jump or incomplete hydraulic jump. Several methods have been proposed for the head loss in step spillways have not taken into account the head loss of the hydraulic jump in the Nappe flow. In the present study the head loss due to the hydraulic jump is also taken into account. Barani (2005) tried to optimize the geometry of the stepped spillway using physical modeling technique.

In addition, ANSYS software has been used to simulate flow over stepped spillways. ANSYS is a finite element model which solves Navier-Stokes and continuity

equations using finite element method. It can simulate turbulence flow with several turbulent sub modules. Sub modules of ANSYS were examined for the best simulation process of the phenomenon. Finally, the NKE turbulent model of ANSYS was selected to simulate the same flow conditions of the physical modeling in this research program.

**RESULTS AND DISCUSSION**

**Physical modelling:** Energy loss between upstream and downstream of the fall before the hydraulic jump can be calculated by the following equation:

$$\Delta H_1 = H_1 - H_2 = h + 1.5 * y_c - y_1 - 0.5 * (\frac{y_c^3}{y_1^2}) \tag{4}$$

derivation of Eq. 4 over  $dy_1$  gives  $y_1 = y_c$  which is equal to  $h$ , the height of fall. Dimensionless equation of Eq. 4 over  $h$  can be expressed as follows:

$$\frac{\Delta H_1}{h} = 1 + 1.5 * (\frac{y_c}{h}) - \frac{y_1}{h} - 0.5 * (\frac{y_c^3}{h y_1^2}) \tag{5}$$

$$\frac{\Delta H_1}{h} = 1 + 1.5 * (\frac{y_c}{h}) - \frac{y_1}{h} - 0.5 * (\frac{y_c}{h})^2 (\frac{1}{(y_1/h)^2}) = f_1(\frac{y_c}{h}, \frac{y_1}{h}) \tag{6}$$

In Eq. 5 and 6  $\Delta H_1/h$  is a function of  $y_1/h$  and  $y_c/h$ . For a  $y_1/h$  with increasing of  $y_c/h$ ,  $\Delta H_1/h$  is increased up to one which gives the  $\Delta H_1 =$  height of the pool is increased. Then  $\Delta H_1/h$  is decreased down to zero. The maximum of these curves are observed when  $y_c/h = 0.1, 0.3$  and  $0.5$ . Figure 2 demonstrates results of Eq. 5.

Another form of Eq. 5 is as follow:

$$\frac{\Delta H_1}{h} = 1 + 1.5 * (\frac{y_c}{h}) - \frac{y_1}{h} - 0.54 * (\frac{y_c}{h})^{1.275} - 1.715 * (\frac{y_c}{h})^{0.45} = f_1(\frac{y_c}{h}) \tag{7}$$

Figure 3 illustrates the relative energy loss against  $y_1/h$  and  $y_c/h$  of Eq. 7. Figure 3 shows that there is a minimum  $\Delta H_1/h$  when  $y_c/h = 0.85$ .  $y_c/h = 0.85$  is the border line for Nappe and Skimming flow. It denotes that with increasing in the flow discharge next to the border line the flow regime will change from one to the other case. Therefore, in a Nappe flow, increase in the flow discharge in the border line when it is near to the Skimming flow, minimum relative energy loss equal to 24.23% is exist.

In the complete hydraulic jump, energy loss can be estimated by Eq. 8 as below:

$$\Delta H_2 = \frac{(y_2 - y_1)}{4 * y_1 * y_2} \tag{8}$$

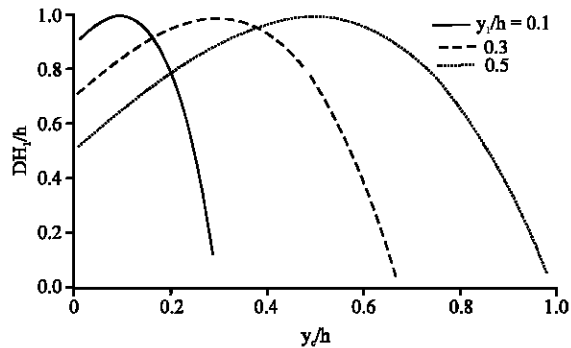


Fig. 2: Illustration of the relative energy loss against  $y_1/h$  and  $y_c/h$

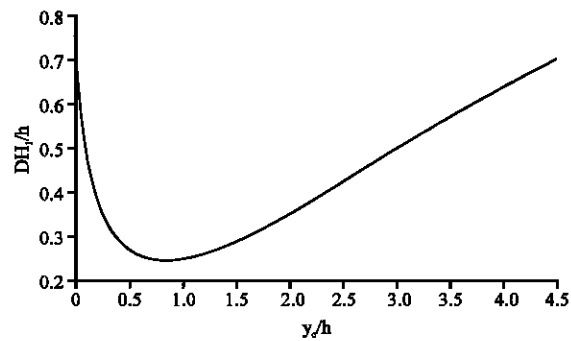


Fig. 3: Demonstration of the relative energy loss against  $y_c/h$

Dimensionless equation of above equation can be as the following equation:

$$\Delta H_2 = \frac{1}{4} * [\frac{y_2}{h} * (\frac{y_2}{y_1} - 3) + \frac{y_1}{h} * (3 - \frac{y_1}{y_2})] = f_2(\frac{y_2}{h}, \frac{y_1}{h}, \frac{y_1}{y_2}) = f' * 2 * (\frac{y_c}{h}, \frac{y_1}{h}, \frac{y_2}{h}) \tag{9}$$

Equation 9 shows that this relative energy is also function of  $y_c/h$ . Figure 4 shows the correlation between these two parameters. It is observed that this relative energy in maximum (0.485) when  $y_c/h = 0.41$ . The energy loss due to the hydraulic jump starts from 0.2 to its maximum with increase in  $y_c/h$ . Then, it will decrease with increase in  $y_c/h$ . The value of  $y_c/h = 1$  is the limit for transformation of Nappe flow regime to Skimming flow regime. The rest of the curve from  $y_c/h = 1$  to  $y_c/h = 4.5$  due to lack of formation of the hydraulic jump is not applicable.

Summation of  $\Delta H_1$  and  $\Delta H_2$  is shown in the Fig. 5. This Fig. 5 gives very good criteria for analysis of the energy loss in the Nappe and Skimming flows. The variation of  $y_c/h$  can be classified to three zones as follows:

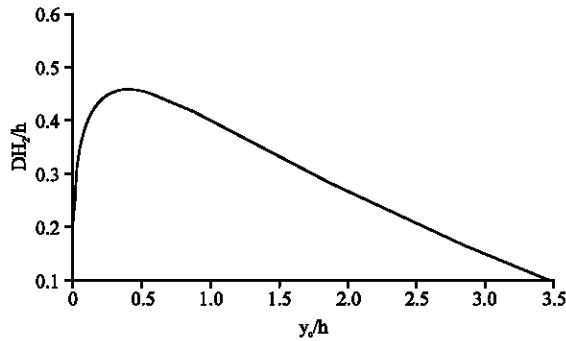


Fig. 4: Variation of the relative energy loss and  $y_c/h$  Eq. 9

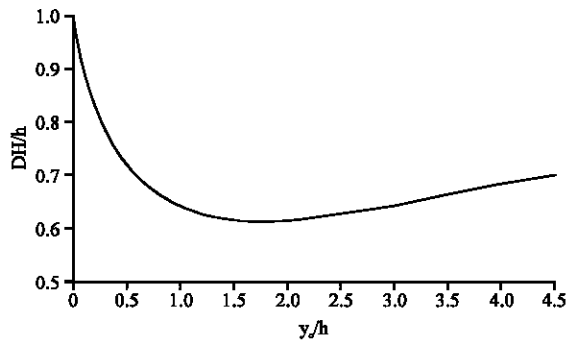


Fig. 5: Summation of relative energy loss against  $y_c/h$

**A:** Zone one for  $y_c/h = 0.0$  to  $1.0$  which in this range, the Nappe flow regime is taken place. In this zone the energy loss is decreased. Comparing with the two other zones in the first zone the energy loss is maximum.

**B:** Zone two for  $y_c/h = 1.0$  to  $1.79$  which in this range the energy loss decrease with increasing in  $y_c/h$ . In  $y_c/h = 1.79$  the relative energy loss equals  $0.618$ . The left limit shows that the nappe flow starts to be established and the right limit of this range denotes the minimum of the energy loss in the Nappe flow regime. Comparison of the first and second zones shows in the Skimming flow regime the head loss is less than the Nappe flow ( $y_c/h = 1$  up to  $1.79$ ). This result is in agreement with findings of Chamani and Rajaratnam (1994) and Matos and Quintela (1994).

**C:** Zone three which  $y_c/h > 1.79$ . In this zone the head loss increases and for the Skimming flow is more than the Nappe flow. There is a good agreement between result of this research program and Chanson results.

Actually, in the long stepped spillways with uniform flow, the head loss in the Skimming flow is more than the Nappe flow. Figure 6 compares findings of some research programs.

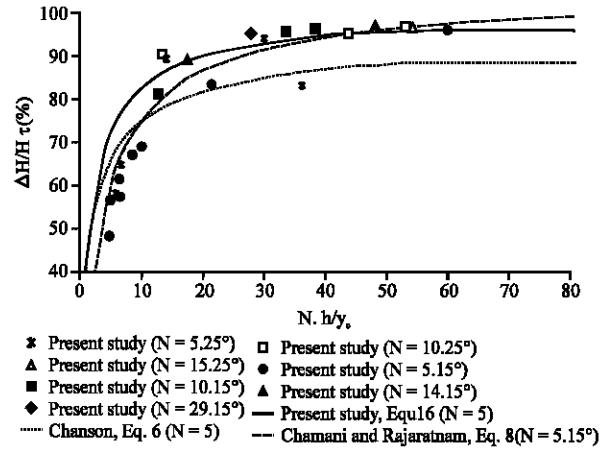


Fig. 6: Comparison of the relative energy loss in different studies

**Mathematical modelling:** ANSYS software has been selected to simulate relative energy loss in the Nappe flow on the stepped spillways. ANSYS is a finite element model which solves Navier-Stokes and continuity equations using finite element method. Flow conditions in the upstream and downstream of the physical models were given to ANSYS as boundary and initial conditions. Figure 7 demonstrates simulated flow velocity vectors over one of physical model established in this study.

As it is shown in Fig. 7, ANSYS software is able to simulate the flow hydraulics over stepped spillways very successfully. Several mathematical models based upon established physical models were developed in ANSYS. Using data of the physical models as boundary conditions, relative head losses due to different number of steps in the spillway are compared with the observed data and the same results of Chanson's proposed experimental equation. Figure 8 shows this comparison for two very important dimensionless parameters in the dimensional analysis of the phenomenon.

As can be seen in the Fig. 8, a very good agreement is seen between observed data and ANSYS outputs. As it is shown, Chanson's experimental equation results are less agreed with the observed data. In addition, the same comparison is undertaken for relative energy loss in the Nappe flow due to number of steps in the stepped spillway. Figure 9 demonstrates the results of this part of the study.

In Fig. 9, Chanson's experimental equation has shown less accuracy than ANSYS. However, there is very good agreement between ANSYS outputs and experimental observation data.

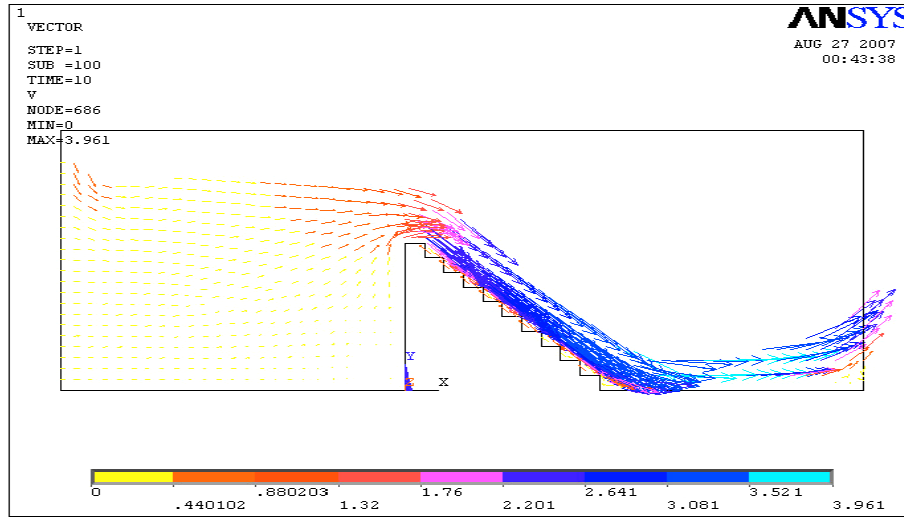


Fig. 7: Predicted velocity vectors for the collected experimental data

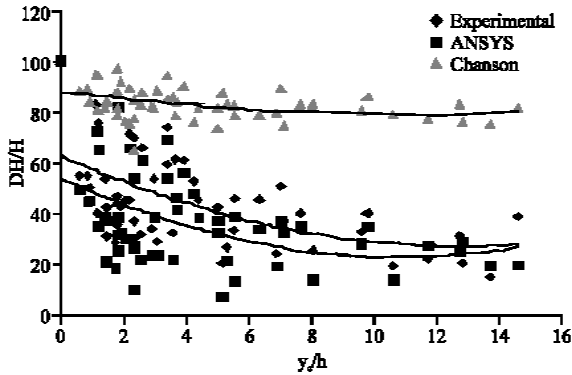


Fig. 8: Comparison of ANSYS outputs and Chanson's experimental equation with observed data for the Nappe flow

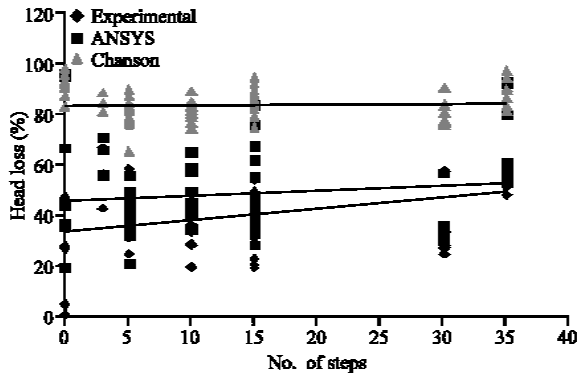


Fig. 9: Comparison of ANSYS outputs and Chanson's experimental equation with observed data for different number of steps

## CONCLUSIONS

In the present study, the Nappe flow hydraulics has been studied at, Shahid Chamran University, Iran using physical and mathematical models. The energy loss in the Nappe flow is classified into two parts and dimensionless equations have been derived for both of them. In the stepped spillways with mild slope, complete hydraulic jump or incomplete hydraulic jump can be formed. Therefore, the energy loss of hydraulic jump has been entered in the equations. Then, for assessment of the accuracy of the equations, several experiments have been set up in the hydraulic laboratory. The experiments have been undertaken in two models with slope angle of 15 and 25°. In the Nappe flow with high discharge ( $Nh/y_c < 15$ ), the equations presented by Chamani and Rajaratnam is in very good agreement with results of this research program.

The hydraulic jump effect is not considered in the presented equations by the researchers in their previous. It denotes that in the Nappe flow with high flow discharge, the hydraulic jump does not form. Therefore, the head loss due to the hydraulic jump is not considered in the previous studies. However increasing of  $Nh/y_c$  in the lower flow discharge, the hydraulic jump is formed over each individual step. It is considered in the present study. Finally, ANSYS is employed to simulate hydraulics over stepped spillways. It is concluded that ANSYS has capability to cope with the phenomenon in the stepped spillway with less than 6% error.

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