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Input-Output Pairing for Nonlinear Multivariable Systems

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Abstract: Decentralized control is a well established approach to control the multivariable processes. In this approach, control structure design and in particular input-output pairing is a vital stage in the design procedure. There are several powerful methods to select the appropriate input-output pair in linear multivariable systems. However, despite the fact that most practical processes are nonlinear, there is no general method to select the appropriate input-output pair for nonlinear multivariable systems. In this study, a new general approach to input-output pairing for linear and nonlinear multivariable systems is proposed. Simulation results are employed to show the effectiveness of the proposed methodology.

Key words: Decentralized control, input-output pairing, nonlinear multivariable systems, relative gain array

INTRODUCTION

Decentralized control methodology provides an effective solution to control complex processes with multi inputs and multi outputs. Simple tuning procedure for single input and single output controllers and implementation considerations are among the many reasons for its wide use (Takagi and Nishimura, 2003; Tan *et al*., 2001; Asano and Morari, 1998; Skogestad and Postlethwaite, 2005).

However, a successful decentralized design would require an appropriate input-output section a prior (Skogestad and Postlethwaite, 2005). Since the seminal work by Bristol (1966) and presentation of RGA concept for input-output pairing, there have been many extensions and modifications to the method (Khaki-Sedigh and Shahmansoorian, 1996; Conley and Salgado, 2000; Wittemark and Salgado, 2002; Xiong *et al*., 2005). In spite of the extensive research of the previous decades, the input-output pairing of nonlinear multivariable systems remains an open problem and only recently it has been addressed. Glad (1999) presents an extension of RGA to input-output pairing for nonlinear multivariable systems. Where, this approach proposes a two stage static and dynamic input-output pairing analysis. Also, Moaveni and Khaki-Sedigh (2006) proposed a new on-line estimation of RGA using neural network for linear, nonlinear or uncertain linear multivariable systems.

INPUT-OUTPUT PAIRING FOR NONLINEAR MULTIVARIABLE SYSTEMS

Input-output pairing for nonlinear multivariable systems is mainly performed by the indirect approach, where the system is linearized around its operating points and any of the approaches for linear multivariable systems are used. Recently, Glad (1999) has presented an extension of RGA to input-output pairing for nonlinear multivariable systems.

In the following theorem a mathematical relationship between inputs and outputs of nonlinear multivariable systems is derived and using it a new input-output pairing analysis based on relative gain definition will be presented.

Theorem, Consider the class of multivariable nonlinear systems, described in affine state space model with m inputs and m outputs, given by:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{j=1}^{m} g_j(x)u_j \\ y_1 &= h_1(x) \\ &\vdots \\ y_m &= h_m(x) \end{aligned} \tag{1}$$

Where, $f(x)$, $g_1(x)$,..., $g_m(x)$ are smooth vector fields and $h_1(x)$,..., $h_m(x)$ are smooth functions, defined on an open set of $\Re^n$, u_j shows the jth input of input vector, u

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and y_i shows the ith output of output vector, y (Isidori, 1995). Input-output pairing analysis for nonlinear multivariable systems can be done using $\Gamma_{nl\text{-}RGA}$ as:

$$\Gamma_{nl\text{-}RGA} \cong R \otimes R^{-T} \tag{2}$$

Where:

$$R = \begin{bmatrix} L_{g_1(x)} L_{f(x)}^{r-1} h_1(x) & \cdots & L_{g_m(x)} L_{f(x)}^{r-1} h_1(x) \\ L_{g_1(x)} L_{f(x)}^{r-1} h_2(x) & \cdots & L_{g_m(x)} L_{f(x)}^{r-1} h_2(x) \\ \vdots & & \vdots \\ L_{g_1(x)} L_{f(x)}^{r-1} h_m(x) & \cdots & L_{g_m(x)} L_{f(x)}^{r-1} h_m(x) \end{bmatrix} \tag{3}$$

and $r \le n$ is the maximum value of vector relative degree from y_i to u_i, where, i, j = 1,..., m.

Proof: As $h_i(x)$ are assumed smooth functions, Taylor series expansion of $y_i(t)$ at point $t = \tau$, can be written in the following form:

$$y_i(t) = \sum_{k=0} \frac{d^k y_i(\tau)}{dt^k} \frac{(t-\tau)^k}{k!} \tag{4}$$

Where:

$$\begin{aligned} \dot{y}_i(t) &= \frac{dy_i}{dt} = \frac{dh_i(x)}{dt} = \frac{dh_i(x)}{dx} \times \frac{dx}{dt} \overset{(1)}{=} \frac{dh_i(x)}{dx} \times \\ &(f(x) + \sum_{j=1}^{m} g_j(x) u_j) = \\ &= \frac{dh_i(x)}{dx} \times f(x) + \sum_{j=1}^{m} \frac{dh_i(x)}{dx} g_j(x) u_j = L_{f(x)} h_i(x) + \\ &\sum_{j=1}^{m} L_{g_j} h_i(x) u_j \end{aligned} \tag{5}$$

so:

$$y_i^{(k)}(t) = L_{f(x)}^k h_i(x) + \sum_{j=1}^{m} L_{g_j(x)} L_{f(x)}^{k-1} h_i(x) u_j \tag{6}$$

therefore, Eq. 4 can be rewritten as:

$$y_i(t) = \sum_{k=0} L_{f(x)}^k h_i(x) \frac{(t-\tau)^k}{k!} + \sum_{k=0} \sum_{j=1}^{m} L_{g_j(x)} L_{f(x)}^{k-1} h_i(x) u_j \frac{(t-\tau)^k}{k!} \tag{7}$$

Where:

$$y_i(t) = \sum_{k=0} L_{f(x)}^k h_i(x) \frac{(t-\tau)^k}{k!} + \left[\sum_{k=0} L_{g_1(x)} L_{f(x)}^{k-1} h_i(x) \frac{(t-\tau)^k}{k!} \quad \cdots \quad \sum_{k=0} L_{g_m(x)} L_{f(x)}^{k-1} h_i(x) \frac{(t-\tau)^k}{k!} \right] \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix} \tag{8}$$

and so:

$$\begin{bmatrix} y_1(t) \\ \vdots \\ y_m(t) \end{bmatrix} = \sum_{k=0} \begin{bmatrix} L_{f(x)}^k h_1(x) \\ \vdots \\ L_{f(x)}^k h_m(x) \end{bmatrix} \frac{(t-\tau)^k}{k!} + \underbrace{\sum_{k=0} \frac{(t-\tau)^k}{k!} \begin{bmatrix} L_{g_1(x)} L_{f(x)}^{k-1} h_1(x) & \cdots & L_{g_m(x)} L_{f(x)}^{k-1} h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1(x)} L_{f(x)}^{k-1} h_m(x) & \cdots & L_{g_m(x)} L_{f(x)}^{k-1} h_m(x) \end{bmatrix}}_{\Phi} \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix} \tag{9}$$

Now, to propose an interaction measure or input-output pairing analysis we can use the relative gain definition (Bristol, 1966). Relative gain is the ratio of the process gain in an isolated loop to process gain in that same loop when all other control loops are closed. Process gain in an isolated loop of process can be computed as:

$$\left.\frac{\Delta y_i}{\Delta u_j}\right|_{\substack{\Delta u_k=0\\ k\neq j}} = \sum_{k=0}^{\infty} \frac{(t-\tau)^k}{k!} L_{g_j(x)} L_{f(x)}^{k-1} h_i(x) = \Phi_{ij} \tag{10}$$

Also, process gain in that same loop when all other control loops are closed is:

$$\left.\frac{\Delta y_i}{\Delta u_j}\right|_{\substack{\Delta y_k=0\\ k\neq i}} = \frac{1}{\Phi_{ji}^{-1}} \tag{11}$$

So, nonlinear RGA, when, t is close to τ, is defined as:

$$\Gamma_{\text{nl-RGA}} = \Phi \otimes \Phi^{-T} \tag{12}$$

Where, the operator $\otimes$ denotes the element-by-element multiplication of the two matrices and -T denotes the inverse transpose of the matrix. However, computation of nonlinear RGA using (12) is not simple, since it requires Φ matrix. So, to propose a practical input-output pairing analysis and to reduce the computational load we can ignore some terms of matrix Φ. These terms includes high order terms (k>r), where, r is the maximum value of vector relative degrees from y_i to u_j (i, j = 1,..., m), because t is assumed close to τ. The low order terms (k<r), where the effect of some inputs on some outputs are not apparent, can be ignored. So, matrix Φ can be approximated as:

$$\Phi \cong \frac{(t-\tau)^r}{r!} \begin{bmatrix} L_{g_1(x)} L_{f(x)}^{r-1} h_1(x) & \cdots & L_{g_m(x)} L_{f(x)}^{r-1} h_1(x) \\ L_{g_1(x)} L_{f(x)}^{r-1} h_2(x) & \cdots & L_{g_m(x)} L_{f(x)}^{r-1} h_2(x) \\ \vdots & & \vdots \\ L_{g_1(x)} L_{f(x)}^{r-1} h_m(x) & \cdots & L_{g_m(x)} L_{f(x)}^{r-1} h_m(x) \end{bmatrix} = \frac{(t-\tau)^r}{r!} R \tag{13}$$

Therefore nonlinear RGA and Eq. 12 can be rewritten as:

$$\Gamma_{\text{nl-RGA}} \cong R \otimes R^{-T} \tag{14}$$

It is readily seen that the nonlinear RGA in Eq. 14 is time independent and has similar properties to the conventional RGA in linear multivariable systems. But $\Gamma_{\text{nl-RGA}}$ is not computed in the steady state and depends on the operating point of the process.

LINEAR INTERPRETATION OF THE NONLINEAR RGA

The above method can be applied to linear multivariable systems. The above method is applied to linear multivariable systems and an interpretation of the nonlinear RGA is provided. Suppose that a linear multivariable system is given by the following state space equations:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix} &= A \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \underbrace{\begin{bmatrix} b_1 & \cdots & b_m \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \\ \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} &= \begin{bmatrix} c^1 \\ \vdots \\ c^m \end{bmatrix} \times \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = C \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \end{aligned} \tag{15}$$

Where:
c^i = ith row of matrix C,
b_j = jth column of matrix B.

According to Eq. 1 and 15, functions f (x), g_j (x) and h_i (x) for the corresponding linear state space model are:

$$\begin{aligned} f(x) &= Ax \\ g_j(x) &= b_j \\ h_i(x) &= c^i x \end{aligned} \tag{16}$$

Equation 13 and 16 are employed to compute the matrix R and to find the relationship between the inputs and outputs of the corresponding linear system. In this case, matrix R is as in Eq. 17 and it is interesting to note that it is similar to the decoupling matrix (Falb and Wolovich, 1967).

$$R = \begin{bmatrix} c^1 A^{r-1} B \\ c^2 A^{r-1} B \\ \vdots \\ c^m A^{r-1} B \end{bmatrix} \tag{17}$$

Hence, using Eq. 17 and 14, a new approach to input-output pairing for linear multivariable systems is provided. Where, it is important to observe that the proposed method takes the full dynamic effects of the system into account, though, in RGA only the steady-state or the behaviour at a single frequency is considered. Hence, the $\Gamma_{\text{nl-RGA}}$ can be used as a dynamic interaction measure for linear and nonlinear multivariable systems, where this method is most effective for nonlinear multivariable systems.

Algorithm: To choose the appropriate input-output pair for affine nonlinear multivariable systems following steps are proposed:

- Calculate the vector relative degrees from y_i to u_j (i, j = 1 ..., m) and assign the largest relative degree to r.
- Compute the matrix R using Eq. 3 (or 17) for nonlinear (or linear) multivariable systems.
- Compute the nonlinear RGA using Eq. 14.
- Choose the appropriate input-output pair using nonlinear RGA analysis.

SIMULATION RESULTS

Nonlinear and linear multivariable processes are used to show the effectiveness of the proposed methodology. The measure of goodness of input-output pairing method is the decentralized control performance (Hovd and Skogestad, 1992). So, in the following examples we use this measure to compare the proposed method with previous methods.

Example 1: Consider the Quadruple-tank with nonlinear state space model as (Johansson, 2000):

$$\begin{aligned}\dot{h}_1 &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1\\ \dot{h}_2 &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u_2\\ \dot{h}_3 &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2\\ \dot{h}_4 &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1\end{aligned} \tag{18}$$

Where, h_1 and h_2 (water levels of 1st and 2nd tank) are outputs of the Quadruple-tank.

Linearization of Eq. 18 gives the following transfer function matrix:

$$G(s) = \begin{bmatrix} \dfrac{\gamma_1 c_1}{1+sT_1} & \dfrac{(1-\gamma_2)c_2}{(1+sT_1)(1+sT_3)} \\ \dfrac{(1-\gamma_1)c_2}{(1+sT_2)(1+sT_4)} & \dfrac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} \tag{19}$$

Where:

$$c_1 = \frac{T_1 k_1 k_c}{A_1},\quad c_2 = \frac{T_2 k_2 k_c}{A_2},\quad T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$$

Assume that the Quadruple-tank has approximately the following physical constants (Johansson, 2000):

$$\begin{aligned}&A_1 = A_3 = 28 \ (\text{cm}^2)\\ &A_2 = A_4 = 32 \ (\text{cm}^2)\\ &a_1 = a_3 = 0.071 \ (\text{cm}^2)\\ &a_2 = a_4 = 0.057 \ (\text{cm}^2)\\ &k_c = 0.50 \ (\text{Vcm}^{-1})\\ &g = 981 \ (\text{cm s}^{-2})\\ &k_i = 2.9\end{aligned} \tag{20}$$

The conventional RGA for the linear model of the Quadruple-tank is:

$$RGA(G) = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix} \tag{21}$$

Where,

$$\lambda = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1}$$

If, $0<\gamma_1+\gamma_2\leq 1$, the appropriate pair is (u_1-y_2, u_2-y_1) and if $0<\gamma_1+\gamma_2\leq 2$ the appropriate pair is (u_1-y_1, u_2-y_2) (Johansson, 2000).

Now to investigate the properties of the proposed nonlinear RGA, we can apply the nonlinear RGA method to this process and analyze the results.

To compute the nonlinear RGA, for this process r = 2 and matrix R is:

$$R = \begin{bmatrix} \left(-\dfrac{k_1k_ca_1\sqrt{g}}{\sqrt{2A_1^2}}\right)\dfrac{\gamma_1}{\sqrt{h_1}} & \left(\dfrac{k_2k_ca_3\sqrt{g}}{\sqrt{2A_1A_3}}\right)\dfrac{(1-\gamma_2)}{\sqrt{h_3}} \\ \left(\dfrac{k_1k_ca_4\sqrt{g}}{\sqrt{2A_2A_4}}\right)\dfrac{(1-\gamma_1)}{\sqrt{h_4}} & \left(-\dfrac{k_2k_ca_2\sqrt{g}}{\sqrt{2A_2^2}}\right)\dfrac{\gamma_2}{\sqrt{h_2}} \end{bmatrix} \tag{22}$$

so, nonlinear RGA is:

$$\Gamma_{nl-RGA} = \frac{1}{\dfrac{\gamma_1\gamma_2}{\sqrt{h_1h_2}} - \dfrac{(1-\gamma_1)(1-\gamma_2)}{\sqrt{h_3h_4}}} \begin{bmatrix} \dfrac{\gamma_1\gamma_2}{\sqrt{h_1h_2}} & -\dfrac{(1-\gamma_1)(1-\gamma_2)}{\sqrt{h_3h_4}} \\ -\dfrac{(1-\gamma_1)(1-\gamma_2)}{\sqrt{h_3h_4}} & \dfrac{\gamma_1\gamma_2}{\sqrt{h_1h_2}} \end{bmatrix} \tag{23}$$

which, it can be rewritten as follows:

$$\Gamma_{nl-RGA} = \begin{bmatrix} \dfrac{1}{1-\dfrac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}\sqrt{\dfrac{h_1h_2}{h_3h_4}}} & \dfrac{1}{1-\dfrac{\gamma_1\gamma_2}{(1-\gamma_1)(1-\gamma_2)}\sqrt{\dfrac{h_3h_4}{h_1h_2}}} \\ \dfrac{1}{1-\dfrac{\gamma_1\gamma_2}{(1-\gamma_1)(1-\gamma_2)}\sqrt{\dfrac{h_3h_4}{h_1h_2}}} & \dfrac{1}{1-\dfrac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}\sqrt{\dfrac{h_1h_2}{h_3h_4}}} \end{bmatrix} \tag{24}$$

To find the appropriate input-output pair from Eq. 24, the following new variables are defined as:

$$\alpha = \frac{1}{1 - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}\sqrt{\frac{h_1h_2}{h_3h_4}}} \tag{25}$$

Hence:

$$\Gamma_{nl-RGA} = \begin{bmatrix} \alpha & 1-\alpha \\ 1-\alpha & \alpha \end{bmatrix} \tag{26}$$

Where, to choose the appropriate input-output pair we should analyze the elements of the nonlinear RGA in Eq. 26. Therefore, the following two cases are defined:

- If $\alpha>1-\alpha$ then the appropriate input-output pair is (u_1-y_1, u_2-y_2).

This result is easily justified as follows:

$$\begin{aligned} &\alpha > 1-\alpha \Rightarrow \quad \alpha > 0.5 \Rightarrow \quad \frac{1}{1 - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}\sqrt{\frac{h_1h_2}{h_3h_4}}} > 0.5 \Rightarrow \\ &\Rightarrow 1+\gamma_1\gamma_2(1-\sqrt{\frac{h_3h_4}{h_1h_2}}) < \gamma_1+\gamma_2 < \min\{1+\gamma_1\gamma_2(1+\sqrt{\frac{h_3h_4}{h_1h_2}}),\ 2\} \end{aligned} \tag{27}$$

Where, it shows that the new result is compatible with the result of conventional linear RGA analysis.

- Also, if $\alpha<1-\alpha$ then the appropriate input-output pair is (u_1-y_1, u_2-y_2). So:

$$\alpha < 1-\alpha \Rightarrow \alpha < 0.5 \Rightarrow \begin{cases} 0<\gamma_1+\gamma_2<1+\gamma_1\gamma_2(1-\sqrt{\frac{h_3h_4}{h_1h_2}}) & \text{(a)} \\ \text{or} & \\ 1+\gamma_1\gamma_2(1+\sqrt{\frac{h_3h_4}{h_1h_2}})<\gamma_1+\gamma_2 \le 2 & \text{(b)} \end{cases} \tag{28}$$

Where, Eq. 28a is compatible with the conventional linear RGA analysis, but, Eq. 28b is additional condition that derived by nonlinear RGA analysis. Therefore, Eq. 27 and 28 show that the appropriate input-output pairs depend on the operating points of nonlinear process.

To compare the input output pairing analysis according to nonlinear RGA with the conventional linear RGA analysis, we consider the following conditions for the Quadruple-tank process:

$$\begin{aligned} &h_1 = 12.7 \ \text{(cm)},\ \ h_2 = 12.4 \ \text{(cm)} \\ &h_3 = 7.4 \ \text{(cm)},\ \ h_4 = 8.7 \ \text{(cm)} \\ &\gamma_1 = 0.48,\ \ \gamma_2 = 0.45 \end{aligned} \tag{29}$$

For the above conditions linear RGA proposes (u_1-y_2, u_2-y_1) as an appropriate input-output pair (Johansson, 2000). Using the nonlinear RGA, following inequality holds true and (u_1-y_1, u_2-y_2) is proposed as an appropriate pair:

$$0.8782<\gamma_1+\gamma_2 = 0.93<1.5538 \tag{30}$$

We use the decentralized control structure with PI controllers introduced by Johansson (2000) to compare the close loop performances of the proposed input-output pairings. In Fig. 1 responses of the Quadruple-tank according to (u_1-y_2, u_2-y_1) are shown and in Fig. 2 the responses according to (u_1-y_1, u_2-y_2) are shown.

Hence, the simulation confirms that the close-loop responses according to diagonal pairing suggested by nonlinear RGA have more desirable performance than the close-loop responses according to off-diagonal pairing. Where, this result shows the advantage of the proposed input-output pairing method.

Example 2: Consider the well-known Wood and Berry binary distillation column process as:

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} \tag{31}$$

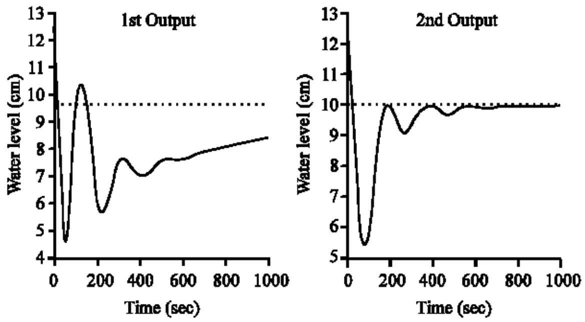

Fig. 1: Output responses according to linear RGA analysis

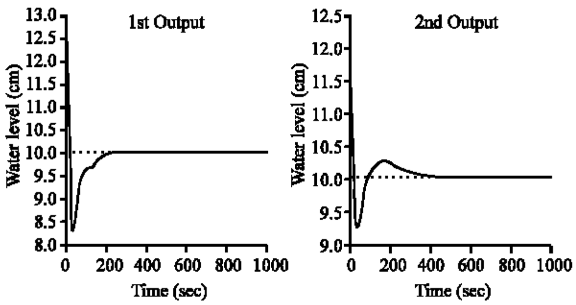

Fig. 2: Output responses according to nonlinear RGA analysis

Where, X_D and X_B are the overhead and bottom compositions of methanol, respectively, R is the reflux flow rate and S is the steam flow rate to the reboiler (Wood and Berry, 1973).

Using Eq. 17 in the above method to choose the appropriate input-output pair, matrix R for this process is as follows:

$$R = \begin{bmatrix} 3.1118 & -1.2429 \\ 0.4016 & -1.8899 \end{bmatrix} \tag{32}$$

and using Eq. 14:

$$\Gamma_{nl-RGA} = \begin{bmatrix} 1.0927 & -0.0927 \\ -0.0927 & 1.0927 \end{bmatrix} \tag{33}$$

Where, a first order Pade approximation to realize the time delays are used and (u_1-y_1, u_2-y_2) is the appropriate input-output pair similar to the conventional linear RGA analysis (Kariwala *et al*., 2006).

Also, Hankel norm based input-output pairing method, using Eq. 34, proposes (u_1-y_1, u_2-y_2) as the appropriate input-output pair.

$$\bar{\Sigma}_H = \begin{bmatrix} 6.7527 & 10.5759 \\ 4.4813 & 11.2749 \end{bmatrix} \tag{34}$$

Example 3: Consider a process given by Grosdidier and Morari (1986) as:

$$G(s) = \begin{bmatrix} \dfrac{5}{4s+1} & \dfrac{2.5e^{-5s}}{(2s+1)(15s+1)} \\ \dfrac{-4e^{-6s}}{20s+1} & \dfrac{1}{3s+1} \end{bmatrix} \tag{35}$$

The conventional RGA implies the (u_1-y_2, u_2-y_1) as the appropriate input-output pair. But, the nonlinear RGA, using Eq. 17 and 14, is:

$$\Gamma_{nl-RGA} = \begin{bmatrix} 1.5244 & -0.5244 \\ -0.5244 & 1.5244 \end{bmatrix} \tag{36}$$

Where, it shows that, (u_1-y_1, u_2-y_2) is an appropriate input-output pair. This loop pairing decision was obtained by Grosdidier and Morari (1986) through analyzing both magnitude and phase characteristics of the interaction between the two loops and by Xiong *et al.* (2005) using the Effective Relative Gain Array (ERGA).

CONCLUSION

In this study, a direct method to input-output pairing for nonlinear multivariable systems is proposed. Also, the implications of this direct method for the linear multivariable case are studied. This introduces the use of decoupling matrix for input-output pairing of linear multivariable systems. The effectiveness of the method for linear and nonlinear multivariable systems is demonstrated by several examples, for which the RGA based loop pairing criterion gives an inaccurate interaction assessment, while the proposed input-output pairing method provides accurate results. Also, this method shows that input-output pairing for nonlinear multivariable systems is dependent on the operating point.

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